# Examination Response

## a) Interpretation of the Regression Line's Slope

The slope of the regression line represents the estimated increase in the amount of nitrogen removed (in parts per hundred) for each one-foot increase in the width of the grass buffer strip. Specifically, it quantifies the rate of change in nitrogen removal efficiency as the buffer strip width increases. A positive slope indicates that wider buffer strips remove more nitrogen from runoff water before it reaches the stream.

## b) Model Prediction Range

I would be cautious about using this model to predict nitrogen removal for buffer strips with widths between 0 and 30 feet for the following reasons:

1. \*\*Extrapolation concerns\*\*: The observed data only covers buffer strips between 6 and 13 feet. Predicting outside this range (especially at 0 feet or up to 30 feet) involves extrapolation, which assumes the linear relationship holds beyond the observed range.

2. \*\*Linearity assumption\*\*: The relationship between buffer width and nitrogen removal may not remain linear throughout the entire 0-30 foot range. Ecological processes often exhibit threshold effects or diminishing returns.

3. \*\*Sample size limitation\*\*: With only eight observations split between two widths, the model estimates have considerable uncertainty, especially at the extremes of prediction.

I would recommend using this model only within the observed range (6-13 feet) and collecting additional data at wider and narrower widths if predictions outside this range are needed.

## c) Sampling Distribution of Sample Mean

The sampling distribution of the sample mean of the observations on nitrogen removal for the four buffer strips with widths of 6 feet would have:

- A mean equal to the true population mean (μ₆) of nitrogen removal for 6-foot buffer strips

- A standard deviation equal to σ₆/√4, where σ₆ is the standard deviation of nitrogen removal amounts for 6-foot buffer strips

- An approximately normal distribution, justified by the Central Limit Theorem, although with only 4 observations this approximation depends on the underlying distribution being roughly normal

## d) Construction of 95% Probability Interval

To construct an interval with probability 0.95 of containing the sample mean of observations from four buffer strips with widths of 6 feet:

1. The interval would be: x̄₆ ± t₀.₀₂₅,₃ × (s₆/√4)

Where:

- x̄₆ is the sample mean for 6-foot buffer strips

- t₀.₀₂₅,₃ is the critical t-value with 3 degrees of freedom (≈ 3.182)

- s₆ is the sample standard deviation of observations for 6-foot buffer strips

From the left graph, I can observe that:

- x̄₆ ≈ 55

- The error bars suggest s₆/√4 ≈ 5

Therefore, the 95% interval would be approximately: 55 ± 3.182 × 5 ≈ 55 ± 15.91, or (39.09, 70.91)

## e) Better Study Plan for Slope Estimation

Based on the plots provided, the second study plan appears to provide a better estimator of the regression line slope because:

1. The error bars (representing standard errors) in the second plot are smaller, indicating less variability in the measurements at each width level.

2. The second design appears to have more data points distributed between 6 and 13 feet (not just at the endpoints), which provides better information about the slope throughout the range.

3. With additional points between the extremes, the second design can better capture any potential non-linearity in the relationship, making the slope estimate more robust.

## f) Alternative Design to Check Linearity Assumption

To check the assumption of a straight-line relationship, I would recommend selecting buffer strip widths that are more evenly distributed across the range of interest. Specifically:

1. Select 8 different widths (e.g., 3, 6, 9, 12, 15, 18, 21, and 24 feet) instead of just two widths.

2. This design would allow for fitting and comparing multiple models:

- Linear regression model

- Quadratic or higher-order polynomial models

- Nonparametric smoothing approaches

3. Statistical tests could then be performed to determine if higher-order terms significantly improve the fit, which would indicate non-linearity.

4. Visual inspection of residuals from the linear model plotted against buffer strip width would also help identify patterns suggesting non-linearity.

This approach provides a more comprehensive assessment of the functional relationship between buffer strip width and nitrogen removal efficiency.